Optimal control describes quadrupedal walking in dogs

Delyle T. Polet[1,*], S. Javad Hasaneini[2], and John E. A. Bertram[3]

[1]Department of Biological Sciences, University of Calgary, Calgary, AB, Canada T2N 1N4 — [2]Sibley School of Mechanical and Aerospace Engineering, Cornell University, Ithaca, New York, USA 14853 — [3]Cell Biology and Anatomy, Cumming School of Medicine, University of Calgary, Calgary, AB, Canada T2N 4N1 — [*]Corresponding Author: dtpolet@ucalgary.ca

Introduction

Quadrupeds typically walk using a symmetrical gait[1] following the sequence Hind-Fore-Hind-Fore with forelimb following hindlimb contact by 15-25% of the stride period[1]. Why is this particular pattern chosen of the many other types of walking gaits available? It is widely held that animals choose gaits that minimize their energy use per unit distance, but it is difficult to compare various gaits due to both the challenge of training animals to use unnatural gaits, and the wide breadth of parameters that can be modified within each footfall sequence.

Optimal control of simple mechanical models is an alternative means to test the energetic hypothesis, as a solution minimizing energetic consumption through leg work can be found quickly without searching through the entire parameter space. We designed a simple planar mechanical model of a quadruped and determined the footfall sequence, kinematics and ground reaction forces that optimized limb work at particular target speeds. We then compared these solutions to those chosen by dogs at the same speeds to determine whether the optimal control program can accurately predict locomotor behaviour.

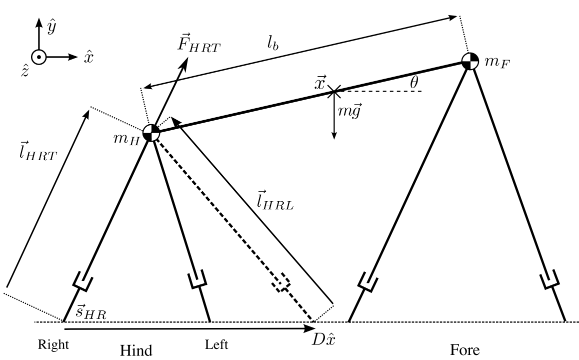

Figure 1: The simple quadrupedal model used in this paper. Two point masses sit on four massless legs that can extend and contract.

Methods

The planar model consists of two point masses, each with two massless, extendable legs. The masses are connected by a rigid trunk (figure 1). Average horizontal speed and stride length are constrained. Additional constraints include periodic motion, above-ground mass position, maximum leg length, equal impulse from left and right limbs, and a single step per limb per stride. The controls are ground reaction forces acting through each limb, and footfall positions are unknown parameters. The objective is the summation of absolute work performed by the limbs. A small penalty is added for the square of the time-derivative of force in order to avoid impulsive forces. Slack variables are used to remedy numerical issues resulting from a non-smooth objective[2]. The bounds on variables are chosen to be biologically realistic and to avoid scaling issues.

Horizontal speed, stride length, fore- and hindlimb length, fore- and hindquarter mass and trunk length for dogs were extracted from [3] and [4], providing inputs to the optimal control problem. The same studies also provided ground reaction force profiles and gait sequences that were compared to the optimal output. The optimal control problem was solved from 30 random initial conditions per input set, using the nlp-solver SNOPT (v. 7.4-1.1)[5] and the transcription and mesh-refinement software GPOPS-II (v. 2.1)[6].

Results

The optimal solution is a lateral sequence walk, with footfall timings that closely match those of real dogs (figure 2). Notably, the optimal gait is symmetrical, a common feature of quadrupedal walking[1]. At a very slow walking speed ($\mathrm{Fr} \equiv \mathrm{u}/\sqrt{\mathrm{g l_F}} = 0.3$), the optimal gait has a lengthy ipsilateral support phase (figure 2a), whereas the same phase is infinitesimally short in dogs (figure 2b). As the center of mass is not above the base of support during ipsilateral stance, dogs may naturally shorten this support phase so as to increase

[1]Duty factors are equal between the left and right fore and hind stances, and the phase offsets between hind or fore footfalls are half the stride period

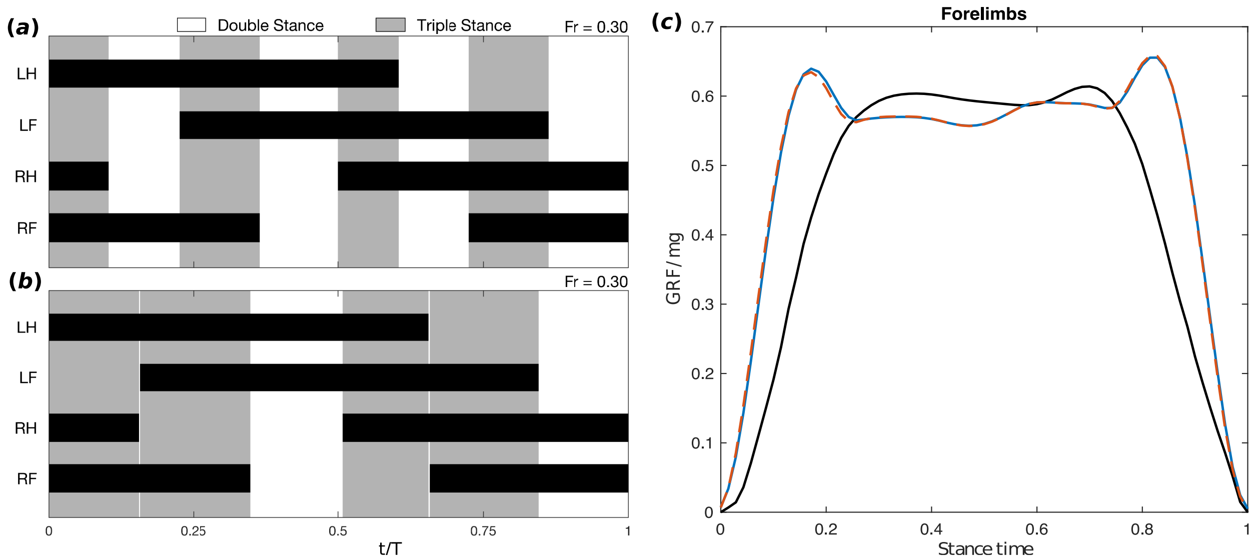

Figure 2: Footfall sequence of the (a) optimal solution and (b) natural gait from [3] at Fr $= 0.30$. (c) Ground reaction forces (GRFs) of the optimal solution (coloured lines) compare well with natural GRFs (black line) as reported by [3].

their stability margins. In the optimal control solution, no stability margin was imposed. The ipsilateral support phase lengthens with faster speeds in dogs[4], and so the agreement of the optimal solutions to natural gaits increases with speed.

Optimal ground reaction forces agree with empirical observation in qualitative shape and magnitude (figure 2c). The optimal control program discovers an M-profile in ground reaction forces, which is commonly observed in quadrupedal walking. However, the simulation overpredicts the magnitude of the peaks.

These results suggest that quadrupedal walking is stereotypical primarily due to its energetic optimality, rather than because of other constraints on the organism. Common features of quadrupedal walking, including gait symmetry, footfall timing and the M-shaped force profile, result from optimizing leg work in a simple quadrupedal model. However, other constraints, such as stability, may play a smaller role in gait choice. These results also show that complex models, including springs and massive legs, are not essential to understanding the general features of quadrupedal walking. Further work will test whether the agreement continues for different morphologies, and whether other gaits, such as the trot and gallop, can be discovered by the same optimal control program at higher speeds.

Acknowledgments

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References

[1] Loscher DM. Kinematische Anpassungen Zur Kollisionsreduktion Im Schritt Vierfüßiger Lauftiere. Freie Universität Berlin; 2015. Title Variation: "Kinematic adaptations to collision reduction in the walking gait of cursorial quadrupeds".

[2] Betts JT. Practical Methods for Optimal Control and Estimation Using Nonlinear Programming. Smith RC, editor. Advances in Design and Control. Society for Industrial and Applied Mathematics; 2010.

[3] Griffin TM, Main RP, Farley CT. Biomechanics of Quadrupedal Walking: How Do Four-Legged Animals Achieve Inverted Pendulum-like Movements? J Exp Biol. 2004;2072(20):3545–3558.

[4] Jayes A, Alexander R. Mechanics of Locomotion of Dogs (*Canis Familiaris*) and Sheep (*Ovis Aries*). J Zool. 1978;185(3):289–308.

[5] Gill PE, Murray W, Saunders MA. SNOPT: An SQP Algorithm for Large-Scale Constrained Optimization. SIAM Rev. 2005;47(1):99–131.

[6] Patterson MA, Rao AV. GPOPS-II: A MATLAB Software for Solving Multiple-Phase Optimal Control Problems Using Hp-Adaptive Gaussian Quadrature Collocation Methods and Sparse Nonlinear Programming. ACM Trans Math Softw. 2013;41(1):1.